Universal Bound on Dynamical Relaxation Time from Condition for Relaxing Quantity to be Classical

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Abstract

It is shown that the Hod's universal bound on the relaxation time of a perturbed system [1] can be derived from a well-known condition for a relaxing quantity to be classical in the fluctuation theory.

In a recent paper [1] Shahar Hod has derived the universal bound on the relaxation time τ of a perturbed system from information theory and thermodynamical considerations,

$$\tau \geq \hbar/\pi T, \tag{1}$$

where T is the system's temperature.

In this note I want to point out that (1) follows from quantum mechanics and thermodynamics directly without any reference to information theory; it can be derived from a well-known condition for a relaxing quantity to be classical in the fluctuation theory.

The proof is given in detail by Landau and Lifshitz [2]. The essential idea can be stated simply. The response of a system in thermal equilibrium to an outside perturbation and the relaxation timescale at which the perturbed system returns to an equilibrium state can be considered in the framework of the fluctuation theory. Landau and Lifshitz have found [2] that if a fluctuating quantity x is to be classical, it must satisfy the condition:

$$\tau \gg \hbar/T, \quad T \gg \hbar/\tau. \tag{2}$$

This is just the Hod's universal bound. But here it is obtained in more fundamental way and in that way, in our opinion, is more appropriate. The condition (2) ensures that quantum effects are negligible in our thermodynamical considerations. But when τ is too small (x varies too rapidly) or when the temperature is too low the fluctuations cannot be treated thermodynamically, and the purely quantum fluctuations dominates.

In other words, the relaxing quantity x behaves classically if its components with frequency ω not satisfying the condition $\hbar\omega \ll kT$ are negligible. The contributions of frequency ω to the dependence x on time appear due to the matrix elements of x between the stationary states differing by $\hbar\omega$. Hence, if the relaxing quantity x is to be classical, it must have negligible matrix elements between states, the energy difference of which is not small as compared with kT.

That is why a typical laboratory system in the Hod's example has the relaxation timescale which satisfies the more stronger condition (2) than (1). It is important to note that an equilibrium state of a system is independent of how and under what conditions this state was reached: due to the thermodynamical or quantum fluctuations. Thus the Hod's bound holds in any case. This is also clear from dimensional arguments.

But in case of a black hole the relaxation time is of the same order of magnitude as the minimal relaxation time, $\tau_{min} = \hbar/T$. In that case, as has been shown above, the quantum fluctuations are large. Does it mean that a relaxing black hole parameter doesn't behave classically? I think that this is not the case. First, as far as is known the black hole perturbations are quite satisfactorily governed by the classical Regge-Wheeler and Teukolsky equations. Secondly, and what is more important, the point is that the time τ need not be the same as the relaxation time for equilibrium to be reached with respect to x, and may be less than this time if x approaches $< x >$ in an oscillatory manner [2]. For example, in case of the variation of pressure in a region of a body with linear dimensions R, τ will be of the order of the period of acoustic vibrations with wavelength $\lambda \sim R$, i.e. $\tau \sim R/c$, where c is the velocity of sound. The quasinormal modes of a black hole have just the same oscillatory character.

References

[1] S. Hod, Phys. Rev. D **75**, 064013 (2007).

[2] L. Landau and E. Lifshitz, *Statistical Physics* (Pergamon Press, Oxford, 1980).